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El reloj de péndulo como introducción a la Automática

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Resumen

El reloj de péndulo puede considerarse como el autómata típico del período pre-cibernético: los engranajes pueden tener sólo una de un número finito de posiciones o estados en cada oscilación del péndulo; cada estado, a través del funcionamiento del escape, que hace uso de realimentación del ángulo, determina el siguiente estado y una salida discreta que se muestra como un cambio discreto en la posición de las manecillas del reloj. Este dispositivo es, por lo tanto, un excelente candidato para servir como un ejemplo introductorio a la automática. En este artículo describiremos brevemente el camino histórico que condujo a la invención del reloj de péndulo, y propondremos un modelo simple del mismo y tres formas de simular su funcionamiento que pueden usarse en un curso básico de automática.

Palabras clave: Péndulo, Escape, Reloj de péndulo, Sistema híbrido, Autómata híbrido.

The pendulum clock as introduction to automation

Abstract

The pendulum clock can be considered as the typical automaton of the pre-cybernetic period: the gears can have only one of a finite number of positions or states in each swing of the pendulum; each state, through the operation of the escapement, which makes use of angle feedback, determines the next state and a discrete output which is displayed as a discrete change in the position of the clock hands. This device is therefore an excellent candidate to serve as an introductory example to automation. In this paper we will briefly outline the historical path that led to the invention of the pendulum clock, and propose a simple model of it and three ways of simulating its operation that can be used in a basic course on automation.

Keywords: Pendulum, Escapement, Pendulum clock, Hybrid system, Hybrid automaton.

1. Introduction

Without being too rigorous, we could say that the ultimate purpose of automation is to replace a living being in a process with artificial devices, the process being able to be intellectual or physical. Aristotle already formulated this purpose in his *Politics*, saying (Aristóteles, 1986, I, 4, 1254a):

> For if every tool could perform its own work when ordered, or by seeing what to do in advance, [...], if thus shuttles wove and quills played harps of themselves, master-craftsmen would have no need of assistants and masters no need of slaves.

The above words also define the two essential aspects of the discipline, the automation of production and the automation of actions, which correspond, respectively, to *industrial automation* and *automata* fields, the last understood in their most general sense. *Control theory* is essential for both of these aspects, and completes the core of current automation.

The pendulum clock, was a notable milestone in the human quest to measure time accurately, and has been considered a paradigm of machines (Aracil, 2010, Chap. 2), being the typical automaton of the pre-cybernetic period: the gears can have only one of a finite number of positions or states in each swing of the pendulum; each state, through the operation of the escapement,

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which makes use of angle feedback, determines the next state and a discrete output which is displayed as a discrete change in the position of the clock hands.

This device is therefore an excellent candidate to serve as an introductory example to automation. In this paper we will briefly outline the historical path that led to the invention of the pendulum clock, and propose a simple model and three ways of simulating its operation that can be used in a basic course on automation.

2. Timekeepers

The oldest timekeeper was the sundial, which provides precision to the simple observation of the everyday sunrise and sunset, as well as of seasonal ascension and declination. Originating in Babylon or Egypt, their trace can be followed through the Fertile Crescent, and the arriving to Greek and Roman cultures is present in the testimony of their writers and historians. Thus Herodotus tells us (Heródoto, 1983, II, 109, 3): «... the Greeks learned the pole, the gnomon, and the division of the day into twelve parts from the Babylonians.»

Probably, humans first learned to estimate the hour by measuring the shadow of a mountain, of a tree or of their own bodies. So, mountain, tree or body were used as gnomons: a vertical structure indicating the time by the length of its projected shadow.

Later, people learned to match the measured shadow according to the season of the year by drawing concentric circles around it, the angle of inclination of the gnomon or stilus to the geographical latitude, and they do all this so that the estimate of the time would be reasonably accurate in any season of the year and in any place on Earth. It was the beginning of a new subject of study, the *gnomonics* or the science of sundials, which combines geometry and astronomy (Jünger, 1998). The simplest sundial consists of a flat plate (the dial) and a gnomon, which projected a shadow onto the dial. As the sun appears to move through the sky, the shadow aligns with different hour-lines, which are marked on the dial to indicate the time of day. The style is the time-telling edge of the gnomon, though a single point or nodus may be used. The gnomon casts a broad shadow; the shadow of the style shows the time. The gnomon may be a rod, wire, or elaborately decorated metal casting. The style must be parallel to the axis of the Earth's rotation for the sundial to be accurate throughout the year. The style's angle from horizontal is equal to the sundial's geographical latitude. With the sundial, humans begin to separate day and night, and, which is more important, to divide them into twelve parts with seasonal varying lengths, the so called *temporal or seasonal hours*: since summer days are longer than winter days, the corresponding summer hours were longer than winter hours (Mayr, 1989).

But the sundial give no information if it is cloudy, and provides no other output. For solving these and other drawbacks, like low precision, a step forward was done with the invention of the water clock or the *clepsydra* (from the Ancient Greek 'to steal water'). The essential elements of the device are a water vessel or container with a slowly changing level that, tracked by a float and transmitted through levers, wheels, gears and linkages, served not only as an element for measuring time, but also as a source or power that could drive sophisticated and large mechanisms.

So, in addition to a greater precision in measuring the time and a total independence of external phenomena, the water clock made two important contributions to our history: it was an *automaton* and a *self-adjusting machine*: «it taught how to regulate machinery over long spans of time by means of program control» (Mayr, 1989, p. 4). And it paved the way for the mechanical clock.

3. The first modern machine

From the beginning, as we have seen, the measurement of time was attempted through a physical phenomenon that took advantage of or emulated some characteristics of time observed in external nature: continuity, uniformity and unidirectionality. Thus, sundials were succeeded by water clocks and hourglasses, making either the water or the sand flow continuously, just as time apparently flows. The invention of the mechanical clock meant oblivion of the cosmic and natural rhythms and the adoption of rigid units to measure time, the change from temporal hours to equinoctial ones (Mayr, 1989). The machine age began when the idea of naively and humbly following nature was abandoned in favor of replicating or replacing it; and use counting instead of observing a course to make the measurement (Aracil, 2010; Landes, 1983).

The mechanical clock has a tangled history, some people arguing that, anticipated by the automated gadgets of the antiquity, including water-clocks, it had a long tradition in Chinese culture before arriving to Europe (Needham et al., 2008; Ronan, 1997). In any case, the device was first driven by weights or springs, and the periodic motion obtained by wheels and controlled by the *escapement*. This mechanism was conceived to regulate the rate of the clock by allowing the gear train to advance at regular intervals.

On the other hand, spring powered clocks supposed an advance over the weight driven ones because they were compact and capable of storing energy for long periods of operation, these two characteristics meaning portability and the possibility of domestic and personal uses, as well as its integration as parts of more complex machines.

But the most perfect realization of the mechanical clock came with the pendulum clock in the 17th century. Galileo already worked on it as a part of his studies about falling bodies (supposing the body moving down a succession of small inclined planes), and was aware of its application as a mechanism for improving the precision of clocks, having the idea of using a swinging bob to regulate the motion of these devices (around 1640). But the definitive contribution was made by Christiaan Huygens in his analysis of circular motion as a combination of centrifugal and centripetal forces. The pendulum was approached as a particular case, and his results in the form of generalized formulas serve to inspire both, practical applications and further studies. In his work *Horologium Oscillatorium* (1673) he gave a complete theory of the machine and concluded that the beat of a pendulum is really isochronous only when the pendulum swings following a cycloidal arc (Usher, 1966).

Subsequently, many refinements were made, the majority of them affecting the escapement mechanism or in order to reduce the size, by scientist and clockmakers, but the essential of the design was preserved (Mayr, 1989; Jünger, 1998; Rooney, 2021; Usher, 1966). Figure 1 shows some milestones in clocks accuracy (Shallis, 1986; Lloyd, 1966).

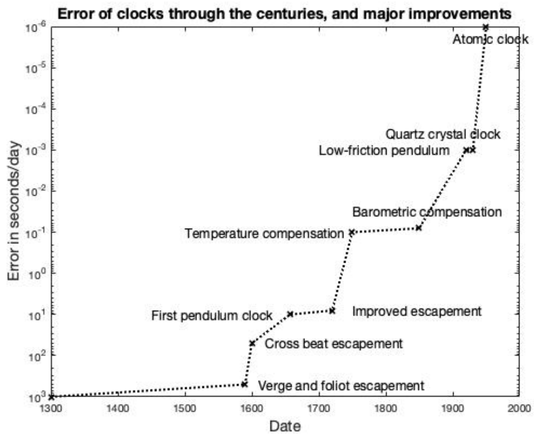

Figura 1: Major milestones in clock accuracy (adapted from (Lloyd, 1966)).

The mechanical clock, which reached its conceptual plenitude in the century of the Scientific Revolution, can be seen as the first modern machine. First, it combined in its design the causality principle of the science then emerging as well as the mechanist philosophy, and inherited the problem-solving and empirical knowledge characteristic of the engineering. Second, it is, at the same time, the most perfect automaton and the most fruitful metaphor of the functioning of the universe until the advent of the digital era. Third, it fuses energy and information, a fact that defines the modern machines. Finally, because the regular measurement of time is the first manifestation of the «application of quantitative methods of thinking to the study of Nature» and helps to believe in «an independent world of mathematically measurable sequences» (Mumford (1979)), the world of the Newtonian science. And so, the clock goes from being an instrument to measure time, natural, telluric or cosmic, and becomes the monarch of time: it produces, governs and organizes a common time to which we must adjust to live in modern society.

4. Alternatives for modeling and simulation

4.1. *Mathematical model*

By using nowadays terminology, the pendulum can be figured out as the two blocks feedback system in Figure 2, one representing the pendulum and the other one the escapement, where $\theta(t)$ is the angle of the pendulum, m the mass and F the force exerted by the escapement mechanism.

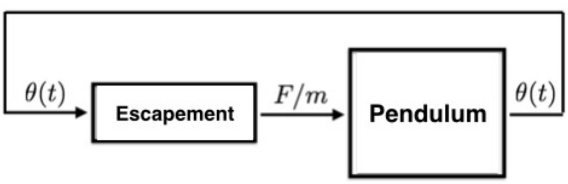

Figura 2: Block diagram of the operation of a pendulum clock with escapement mechanism.

The equation of motion can be formulated as:

$$\frac{d^2\theta(t)}{dt^2} = -\frac{c}{ml^2}\frac{d\theta(t)}{dt} - \frac{g}{l}\sin(\theta(t)) - \frac{F}{m}, \tag{1}$$

being l the length, c the coefficient of friction, and g the acceleration of gravity. The operation of the escapement mechanism can be formulated, in a general way, as follows:

$$F/m = \begin{cases} K\mathrm{sign}(\theta(t)), & \text{if} \quad |\theta_m| \leq |\theta(t)| \leq |\theta_M| \quad \& \frac{d|\theta(t)|}{dt} < 0 \\ 0, & \text{other} \end{cases} \tag{2}$$

being $\mathrm{sign}(\cdot)$ the sign function, and $[|\theta_m|, |\theta_M|]$ the interval of angles where the escapement actuates.

We all have the happy experience of pushing a child on a swing, where, perhaps without realizing it, we function as an escapement. If we want to maintain the amplitude of the oscillation we must do two things: push him at the right time (when the descent begins in the case of the swing), and adjust the force so that it is the strictly necessary to overcome the friction of both the pivot and the body into the air, and to complement the components of the weight that naturally help or hinder the movement depending on its direction. These two actions are represented in our model, respectively, by the escapement operating condition (equation (2)) and by the gain (K) that weights the output.

4.2. *Classical Simulink model*

A complete block diagram ready for simulation using MATLAB® and Simulink® software is presented in Figure 3 (Schwartz and Gran, 2001). The detail of the escapement block is given in Figure 4, and the outputs of both, the angle of the pendulum and the force exerted by the escapement are presented in Figure 9.

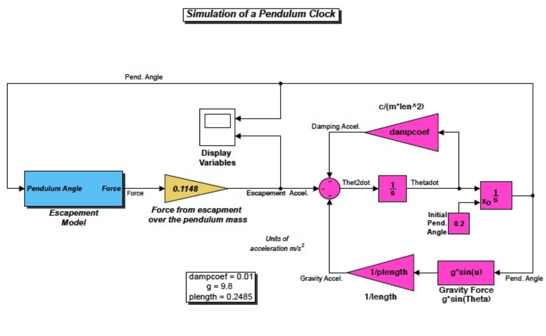

Figura 3: Pendulum clock simulation model.

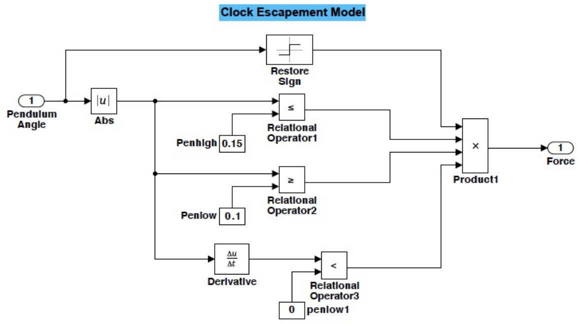

Figura 4: Escapement simulation model.

4.3. *Using finite-state machine for escapement*

Alternatively, we can implement only the escapement mechanism as a finite-state machine or finite-state automaton using the Stateflow tool. In this case, the two discrete states correspond to those in which the escapement is actuating or not. We can see this implementation in Figure 5 and Figure 6.

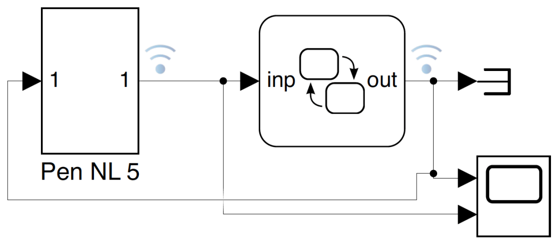

Figura 5: Pendulum clock simulation model with escapement as state machine.

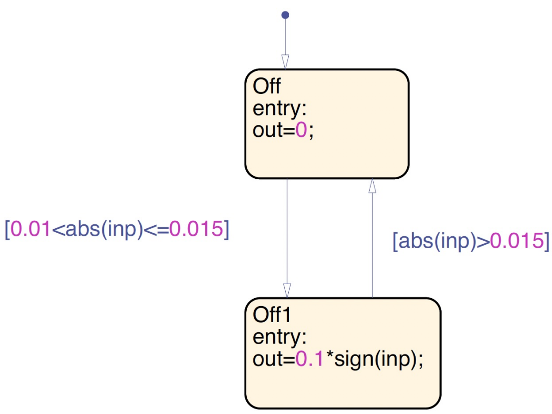

Figura 6: Escapement simulation model as state machine.

4.4. *Modeling the clock as hybrid automata*

Finally, we can utilize the full potential of the Stateflow tool and implement the complete pendulum clock, considered as a hybrid system, using the hybrid automata approximation.

Hybrid systems are dynamical systems that include the interaction of different types of dynamics. We are interested in the interaction of dynamics corresponding to continuous states and discrete states, thus including discrete jumps and continuous flows in the evolutions of their states. The mechanical clock is a paradigmatic case, since it has a continuous motion that is abruptly altered by the action of the escapement.

One of the languages used for modeling hybrid systems is the hybrid automata, finite-state machines with a finite set of continuous variables whose values are described by a set of ordinary differential equations. Formally it is defined as follows (Lygeros, 2004):

Hybrid automaton: *A hybrid autimaton is a collection* $H = (Q, X, f, I, D, E, G, R)$*, being:*

- $Q = \{q_1, q_2, ...\}$ *is a set of* **discrete states**;
- $X = [x_1 x_2 ......x_n]^T \in \mathbb{R}^n$ *a set of* **continuous states**;
- $f(.,.) : Q \times X \to \mathbb{R}^n$ *is a* **vectorial field**;
- $I \subseteq Q \times X$ *is a set of* **initial states**;
- $D(.) : Q \to P(X)$ *is a* **domain**;
- $E \subseteq Q \times Q$ *is a set of* **edges**;
- $G(.) : E \to P(X)$ *is a* **guard condition**;
- $R(.,.) : E \times X \to P(X)$ *is a* **reset map**.

$P(X)$ is the set of all subsets of X. The function $D(q) \subseteq \mathbb{R}^n$ assigns a set of continuous states to each discrete state $q \in Q$. The pair $(q, x) \in Q \times X$ is called a *state* of H. The hybrid automaton defines the possible evolutions for each state. Starting from the initial state $(q_0, x_0) \in I$, the continuous state x evolves according to the differential equation

$$\dot{x} = f(q_0, x), \quad x(0) = x_0, \tag{3}$$

while $q(t) = q_0$. The continuous evolution can proceed as long as x remains in $D(q_0)$. If at some point the continuous state x reaches the guard condition $G(q_0, q_1) \subseteq \mathbb{R}^n$ of some edge $(q_0, q_1) \in E$, the discrete state must change to q_1. At the same time the continuous state is reset to a value in $R(q_0, q_1, x) \subseteq \mathbb{R}^n$. Representing an automaton as an oriented graph (Q, E) with nodes Q and edges E, to each node $q \in Q$ we associate a set of initial states $\{x \in X \mid (q, x) \in I\}$, a vector field $f(q, .) : \mathbb{R}^n \to \mathbb{R}^n$ and a domain $D(q) \subseteq \mathbb{R}^n$. A transition $(q, q') \in E$ starts at q and ends at q'. With each transition $(q, q') \in E$ we associate a guard condition $G(q, q') \subseteq \mathbb{R}^n$ and a restart function $R(q, q', .) : \mathbb{R}^n \to P(\mathbb{R}^n)$.

In our case, the formulation is the following:

- $Q = \{q_1, q_2\}$ (set of discrete states, escapement off and on);
- $X = [x_1 x_2]^T \in \mathbb{R}^2$ (set of continuous states, angle and velocity);
- vector fields

$$f(q_1, x) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -c x_2 - g \sin(x_1) \end{bmatrix},$$

$$f(q_2, x) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -c x_2 - g \sin(x_1) + K \mathrm{sign}(x_1) \end{bmatrix};$$

- $I = \{q_1, q_2\} \times \{x \in \mathbb{R}^2 \mid x_1 \geq 0 \wedge x_2 = 0\}$ (start with initial angle fixing the amplitude of the swing);

- $D(q_1) = \{x \in \mathbb{R}^2 \mid \mathrm{abs}(x_1) > \theta_M\}$ and $D(q_2) = \{x \in \mathbb{R}^2 \mid \theta_m < \mathrm{abs}(x_1) \leq \theta_M\}$ (angle intervals in which the escapement actuates);
- $E = \{(q_1, q_2), (q_2, q_1)\}$ (possibility of switching between on and off and vice versa);
- $G(q_1, q_2) = \{x \in \mathbb{R}^2 \mid \theta_m < \mathrm{abs}(x_1) \leq \theta_M\}$ and $G(q_2, q_1) = \{x \in \mathbb{R}^2 \mid \mathrm{abs}(x_1) > \theta_M\}$ (switch off the escapement outside the fixed angle intervals);
- $R(q_1, q_2, x) = R(q_2, q_1, x) = \{x\}$ (the state is not changed by the switch).

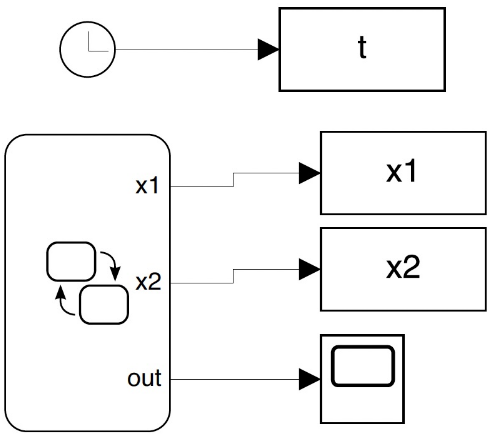

Figura 7: Pendulum simulation model as hibryd automata – General scheme.

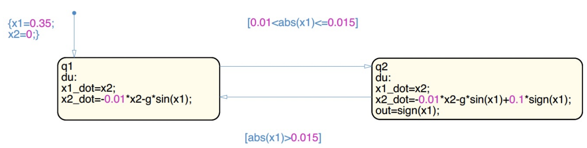

Figura 8: Pendulum simulation model as hibryd automata – State chart.

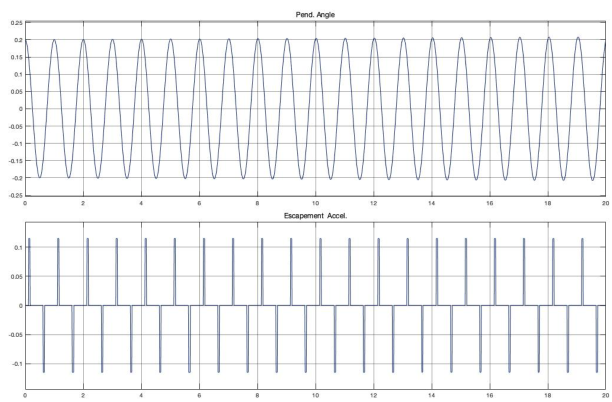

Figura 9: Results of the simulation model.

5. Conclusions

In this paper, the pendulum clock has been presented as an example for introducing automation. The historical path that led to its invention has been outlined, something that is usually neglected, wasting its motivational capacity for students. A simple mathematical model has been used and three simulation models have been proposed: 1) direct translation of the model equations into Simulink blocks; 2) simulation of the escapement using a finite-state machine; 3) simulation of the complete system using the hybrid automaton paradigm.

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