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Kinematic and Dynamic Analysis of Old Mechanism by Modern Means

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Abstract: *This paper presents a procedure to refresh the value of past mechanisms in modern technology by analyzing their performance through modern means of simulation. A consideration of past achievements can be useful to reevaluate past designs by looking at the properties that were used for their development, through modern means of analysis. Examples are illustrated as referring to linkage designs that were proposed by Lorenzo Allievi in 1885.*

Keywords: Classical Kinematics, Analysis of mechanisms, Simulation techniques

I. Introduction

Often in the world of research the scientists tend to project the look forward, that is, to try to find innovation in any way. But one may want to also look back and understand the evolution of technology today. Thus, not only will have a broader perspective about what is done, but moving in a direction more suitable. The concept of technology reaches such a dimension that is even used to classify the different stages of history [1]. For centuries, inventors of all over the world have contributed to develop new and more sophisticated machines. They have enabled us to live a better quality of life. Maybe the documentary heritage found in this regard was not as wide for not treated as an art, but numerous authors have been responsible for Historical rescue of the machines of antiquity. Some of these authors analyze the suitability of the consideration as goods of cultural heritage of Theory and Procedures for Mechanisms [2]. This is the main idea that involves this work. So it is not only a simple analysis but also a tribute and a recognition task to some of that genius. Moreover a review of the different approaches to solve problems related with mechanisms and machines is presented. It can be seen how they have been evolved from the classic methods to the current computer-based methods. In this context, mechanical engineering is the first of the industrial engineering branches, and evolves along with the others [3]. So many of the artifacts whose design is part of consolidated other disciplines, in its first version were mechanical, such as the Watt's centrifugal governor.

Analyzing the history shows that the evolution of human beings both cultural and economic or social, has also been a cause and consequence of the development of technology [1]. Normally, political and economic hegemony has fallen in most developed cultures in technology. This is one of the reasons of the protectionism around inventions. This behavior has occurred since the time of the Egyptians, leaving even today some enigmas, as the type of machinery used in the construction of the Pyramids.

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Sometimes this latter phenomenon is positive for the introduction of competition in certain sectors, but others can be a redundancy of effort, time and money unnecessarily. This is one of the reasons that the project comes ThinkMOTION [4], that as part of Europeana (digital library of freely available digital content more widely in the world), is responsible for the content in the field of kinematics and dynamic domains essential for all technicians. The project ThinkMOTION helps to make a significant improvement in the quantity and quality of digital content and provides a new type of digital library Europeana, with emphasis on technical expertise. It covers both the cradle of humanity expertise as rapid technical progress.

II. Evolution of the design of machines and mechanisms

Mechanisms exist even before the appearance of man. In case of being these mechanisms attributable to any inventor, it would be nature. Thus, in the first vertebrates can be seen that certain mobility problems are solved through the development of legs as they evolved from one species to others [1]. But perhaps the traps to hunt are the first machines built by human. The wheel, water lifting machines, handles machine to turn spears or fire are also inventions of man from pre-history. As a consequence of the Chinese script, in large measure, there is a technological dominance of this culture versus Occidental until the fifteenth century [1]. Textile machinery, catapults, clocks, automatism and infinite hydraulic machines are built during this time. In the Occident highlights the School of Alexandria (from the third century BC). Its advanced culture and technology is combined with the Romans, who develop applications to fields such as civil works (roads, bridges, buildings, etc.) or military (war machines, defense structures, etc.).

In the Middle East there is a strong technological development in the medieval period. In 1206 Al-Jazari includes a large number of machines of his time in his book "Treatise of knowledge about mechanisms" such as fountains, clocks, water wheels or robots [1]. The ability to disseminate information that brings the printing press and the Renaissance movement of the 14th, 15th and 16th centuries put Western Europe at the forefront in the design and development of machines. This movement acquires a considerable presence in Italy [5]. Machines starts to be considered also from an academic point of view, as demonstrate the first course given by Galilei [6]. In the modern age, the technological supremacy over the rest of the world is reflected with the phenomenon of European geographic expansion. In this sense, the Mechanical Engineering plays a fundamental role, and is very present in shipbuilding, armaments, hydraulic machinery, etc. "The Twenty-One Books of the Devices and Machines", by Juanelo Turriano is almost a complete revision of the mechanical artifacts known and practiced in the sixteenth century [1]. James Watt (1736-1819) developed the model of steam engine that lays the foundation of automated work of the Industrial Revolution. Its many applications (textile machinery, railway, automobile, machine tools, etc.) cause economic and social development without precedent [1]. This period coincides with the foundation of school of engineering as the École polytechnique in France. The modern way of teaching and learning of Theory of Machines and Mechanism as we know nowadays begins to appear. For the first time in the history, the Kinematics arises as a consolidated discipline, boosted for the support of mathematicians and mechanical engineers. This phenomenon occurs during the second half of the 19th century. In this context the influential figures of Lorenzo Allievi and Ludwig Burmester play an important role [7,8].

A classification of the methods used for kinematic analysis of linkages maybe done according to the solving procedure used. Thus, three different methods can be distinguished: graphical, analytical and numerical methods [9]. On the one hand, the grapho-analytical methods are based on a geometrical approach that leads to a graphical or analytical solving procedure. On the other hand, analytical methods are based on the equations that define the position of the elements of each mechanism. These approaches are not general purpose methods because the position equations must be derived for each mechanism. Some methods that belong to this group are dyadic decomposition methods, interpolation methods or other that use the modular approach. Lastly, numerical methods facilitate the complete and automatic kinematic analysis of a mechanism. These methods are related to the Multibody Dynamics discipline. When modeling multibody systems (Fig. 1) it is precise to select the coordinates that will unequivocally define the position of each element of the mechanism. The commonly used kinds of coordinates are relative, Cartesian and natural Coordinates.

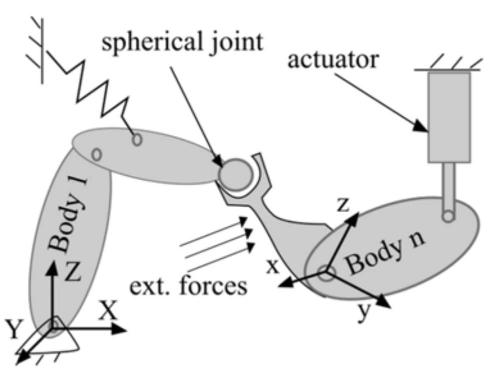


Fig. 1. Example of multibody system

The first approach in this paper addresses the utilization of classical procedures for solving kinematic problems. This involves analytical methods, and, in a larger portion, graphical methods. Concepts as relative velocities and accelerations or instantaneous center of rotation are taken into account. Thus, a kinematic and dynamic analysis of a mechanism may be carried out. Aspects as the study of the inflection circle or the cubic of stationary curvature are difficult to be found on the current literature about Theory of Machines and Mechanisms. However, these issues were often used in the past, as will be shown in the attached problems presented in section 3.

A. The inflection circle

The inflection circle is the circle in which each point has an infinite radius of curvature at a determinate instant and its conjugate point lies at infinity [10]. It can be constructed by a graphical method. Let consider the four-link mechanism be O_A, A, B and O_B as shown in Fig. 2.

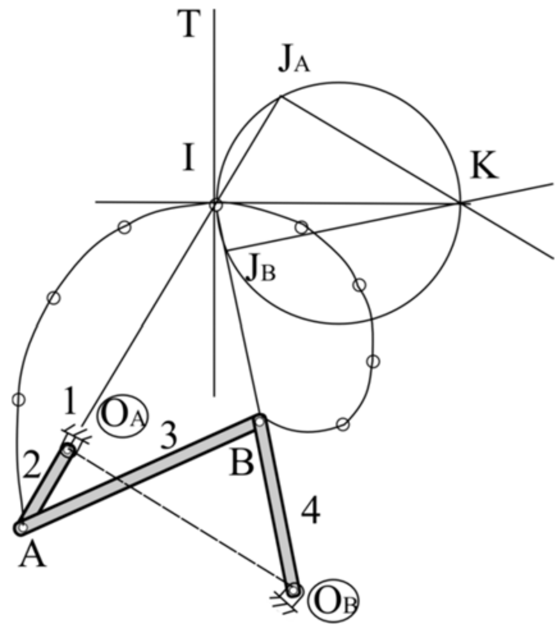


Fig. 2. Inflection circle and cubic of stationary curvature

Now, as the link O_AA can rotate about OA only, therefore, OA is also the conjugate of A with a constant radius of curvature OAA. Thus, A lies on the cubic curve. Similarly, B also lies on the cubic as it has a constant radius of curvature. The procedure is as follows: (1) Locate the instantaneous center I from the intersection of segments O_AA and O_BB, or, if necessary, its prolongations. That is a point of the inflection circle. (2) OA is the center of curvature of the path of the coupler point A. Point J_A will be a second point of the inflection circle. It is placed above the O_AA segment, or its prolongation, and it is obtained from the Euler-Savary equation in the form $\frac{O_AA}{IA} = \frac{IA}{J_AA}$ and $\frac{O_BB}{IB} = \frac{IB}{J_BB}$. (3) Similarly, J_B, a third point of the inflection circle, is calculated following the same procedure. (4) Finally, once three points of the inflection circle are known, its construction is possible. The center G is the intersection of the bisectors of IJ_A and IJ_B.

B. *Cubic of stationary curvature*

Usually, the coupler curve (the locus or path of a point on the coupler) is a sixth-order curve whose radius of curvature changes continuously [11]. However, it is observed that in certain situations, the path has a stationary curvature. Thus, if R is the radius of curvature and s is the distance traveled along the path, then $dR/ds=0$ indicates a stationary curvature of the curve. The locus of all such points on the coupler which have stationary curvature at the instant is known as the cubic of the stationary curvature or the circling-point curve. Stationary curvature does not mean only a constant radius, but also that the continuously varying radius passes through a maximum or minimum value. So, an important property of this curve is that all the paths follows by its points are suitable to for circular and rectilinear trams. That makes this curve to be a good support to synthesis problems [12,13]. Its characteristic equation is:

$$\frac{1}{M \cdot sin\varphi} + \frac{1}{N \cdot cos\varphi} = \frac{1}{r} \tag{1}$$

where φ is the counterclockwise angle from a point of the cubic of the stationary curvature to the oriented line *IA* [11]. For its construction, first is necessary to determine the direction of the common tangent *IT* to the centrodes and 90° rotation of its diameter *IK*. The instantaneous center *I* and the centrode tangent *IT* are the references with respect to which the cubic of stationary curvature is built in polar coordinates by equation (4).

Moreover, points *A* and *B* are on the cubic since their tracks true circles. So, *M* and *N* can be obtained by resolving a set of two equations with two unknown variables. Thus, the construction of the Cubic of stationary curvature may be completed by filling a table with different values of r for a set of values of φ between 0 and 180 degree.

III. Modern simulation techniques

Traditional methods for the kinematic and dynamic study of mechanism have been presented. They are based in principles and theorems of classic Mechanics. Although these methods allow solve easy kinematic and dynamic problems, the complexity of equations involved limits seriously its field of application. Graphical methods sawn have several limitations. On the one hand, they present certain complexity in solving the resultant equations. On the other hand, and more important, there are problems which cannot be solved by these methods. In cases where relative velocities method is not useful, others as auxiliary point method or Goodman's method may be used, but neither have they guaranteed solutions in all cases. Moreover, when dealing with spatial mechanism as the one presented in Fig. 1, it would not be a good idea to use this approach [14].

Analytical methods, in general, are preferred when the problems are relatively affordable. However, when the problems are more complex, is necessary to actualize and adapt the Theory of Machines in order to take advantage of the powerful capabilities of current PC in statement and resolution of equations.

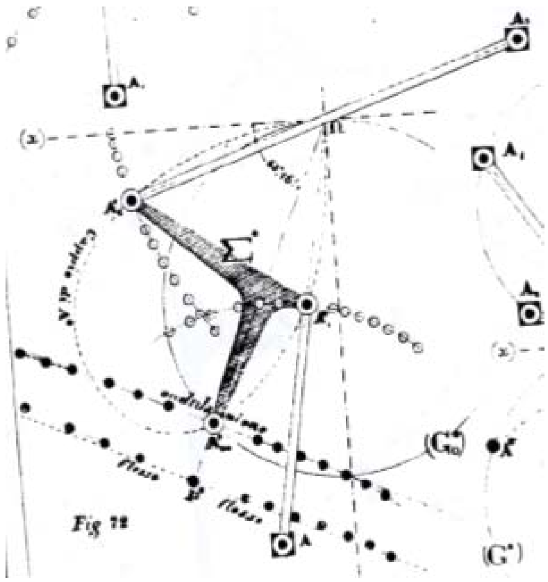
For developing a mathematical programmable model on a computer it is necessary to transform the concepts of link, kinematic pair or velocity among others into a set of numerical data. These data will be disposed as matrices and vectors [15,16]. As seen in part 2, the different analytical methods for kinematic analysis of mechanism can be classified according to the type of coordinates chosen to formulate their constraints and determine their configuration. Some formulations use a large set of absolute coordinates. [17] The position and orientation of the rigid links in the mechanism are described with respect to the global reference coordinate system. The algebraic equations of constraints are introduced to represent the kinematic joints that connect the rigid bodies. Although in this type of formulation the constraint equations are easy to construct, it has the disadvantage of the large number of defined dependent coordinates. Other formulations use sets of relative coordinates [18]. The position of each link is defined with respect to the previous link by means of relative joint coordinates that depend on the type of the joint connecting the two links. His type of formulation yields a minimal set of algebraic equations. The constraint equations are derived based on loop closure equations, and the resulting constraint equations are highly non-linear and contain complex sinusoidal functions.

Another formulation which is based on point coordinates consist in the configuration of the system is described in terms of the rectangular Cartesian coordinates of some defined points in the links and the joints. [17, 18] The system constraint equations are then written to fix the relative positions of the points in each rigid link and also the relative positions between the different links determined by the type of joint connecting them.

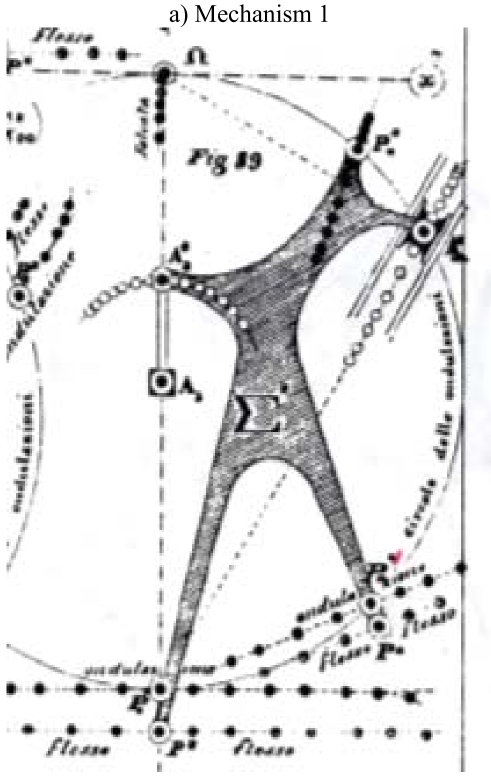
Regarding to the implementation of these techniques on commercial software, in the 70s and 80s, a high development of programs is produced due, in part, to automotive industry. In these applications, is habitual to work with close chains, such as a suspension or steering system. At that time, appear new formulations which are considered the origin of the first commercial multibody software of general purpose. Thus, Sheth and Uicker presented in 1972 the IMP (Integrated Mechanism Program), based on relative coordinates [19] and on matrix transformations. However, Orlandea et al. [20] made a great contribution to global methods. They applied sparse matrix techniques to the dynamic equations and constraint equations from the reference point coordinates. They also used Euler's angles. A bit time later, MSC.Adams [21] was developed from these works. This software is still today the most used in mechanical engineering. In the early eighties, Wehage and Haug [22], and Nikravesh et al. [23] established the theorical fundaments of what eventually become on the DADS software, at University of Iowa. Nowadays it is commercialized by LMS Company as LMS Virtual.Lab Motion [24]. In 1990, Schiehlen [25] presented a catalog with the main characteristics of all the existing mechanism analysis software.

Currently, a huge variety of commercial software for analysis of Multibody Systems can be found. Some of them are Maplesim [26], based on the linear graph theory [27] and Modelica [28], SolidWorks-Cosmosmotion [29] which allows use an MSC.Adams engine on Solidworks environment, or Simmechanics [30] by the Mathworks

Company, based in the relative coordinates modeling approach.



a) Mechanism 1



b) Mechanism 2

Fig. 3. Mechanisms extracted from the Allievis's collection

IV. Cases of study

In order to synthetize the concepts previously explained, this section deals with the resolution of two mechanisms by using modern means. The attached problems have been inspired in a couple of pictures that were rescued from a treatise dating from 19th century about Kinematic of planar motion belonging to Lorenzo Allievi [31]. These pictures are presented in Fig. 3. As can be seen, the concepts of inflection circle and cubic of stationary curvature that have been exposed in section 2 were employed for their kinematic resolution.

The first problem (Fig. 3 (a)) deals with the estimation of the coupler-point curve of a planar four-bar mechanism. Among the different approaches presented in section 3, it has been selected two commercial packages, in order to check its accuracy. Therefore, Simmechanics is employed for determine the grade of approximation of the primitive calculations made by Allievi. Then, a brief dynamic analysis is carried by using the commercial software Simmechanics. The fundamentals of Simmechanics are based in a block-diagrams modeling tool, presenting an interface as depicted in Fig. 4.

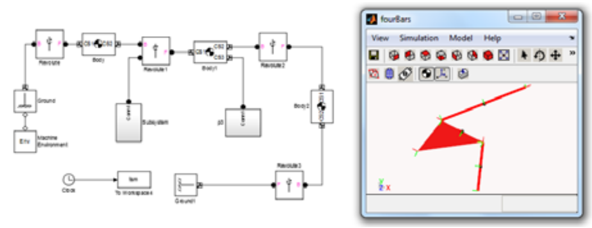


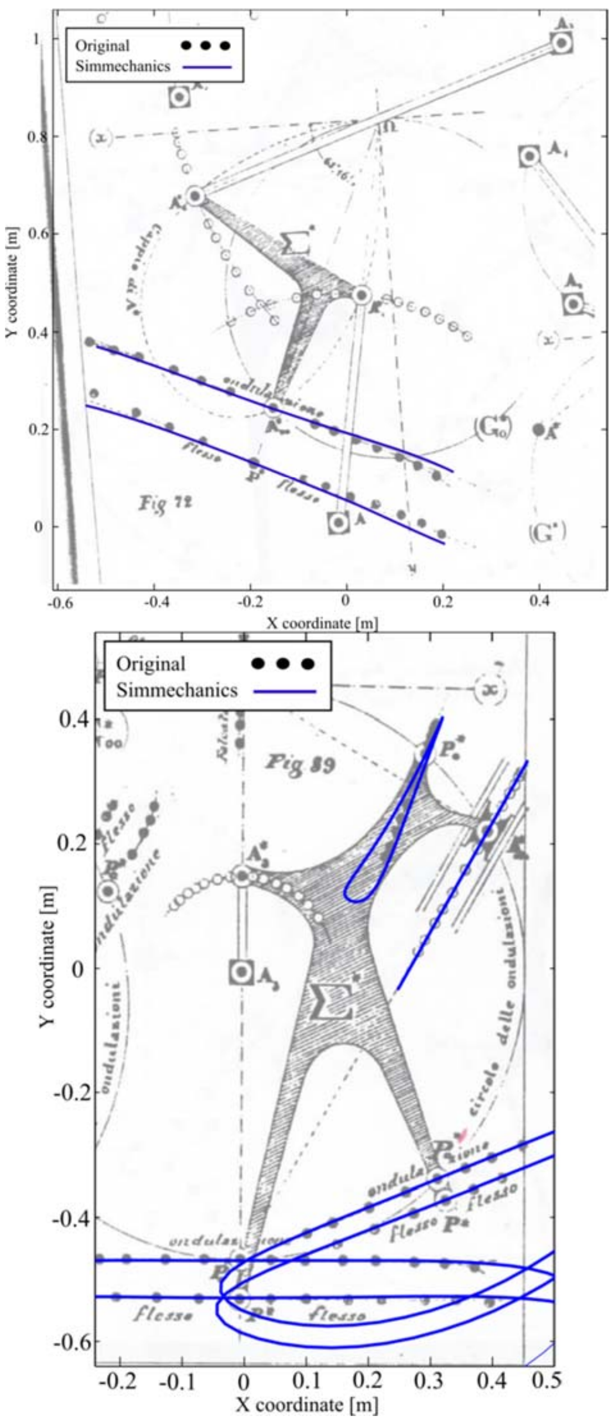
Fig. 4. Simmechanics block diagram interface

This tool is fully integrated with Matlab/Simulink environment, which leads to an easy implementation of algorithm of control or the implementation of user-defined functions. This fact allows taking advantage as the same time of the powerful of calculation presented by Matlab, and its intuitive programming even for non-expertise programmers. Each model is basically modeled by connecting blocks which represents bodies, joints, sensors and actuators. In the definition of a body, it is precise to define its mass and inertia tensor with respect to the body center of gravity, as long as the reference frames corresponding to the points employed on its modeling stage. Then a body is connected to its follower by means of a joint block. External forces of motion conditions are imposed by the actuator blocks. Finally results are generally obtained by the use of sensors attached to the joints and bodies, depending on the kind of information in which the analyst is interested.

A. *Kinematic analysis*

In the kinematic analysis of linkage 1, three body blocks has been employed. The first and the last one are connected to the ground, and represent the crank and the rocker respectively. The middle block is the coupler link, which contains the coupler points whose path is intended to be calculated. All the bodies are connected by revolute joints. The coordinate selected as degree of freedom is the angular position of the crank. By means of an actuator block attached to the first revolute joint, an inner constant angular speed of 10 degrees$\cdot$s^{-1} is imposed. Results presented in Fig 5 (a) show the movement registered by a sensor attached to the coupler body which is defined by the coupler-curve of two of its points. As observed, the path followed by these points predicted by Allievi is not far enough from the exact solution obtained by computational modern methods.

The kinematic analysis of the second mechanism follows the same procedure, with the particularity of the substitution of a revolute joint by a prismatic restriction, and the incorporation of other four coupler points. Results of this analysis are presented along with the original problem, and can be seen in Fig 5 (b).



b) Results for mechanism 2

Fig. 5. Path followed by the points of interest of both mechanisms. The blue lines correspond to a matlab plot representing the Simmechanics kinematic simulation outputs.

B. Dynamic analysis

In the kinematic analysis, no consideration regarding the mass and inertia properties of the linkage has been stated. That means that any arbitrary values have been employed in the mechanism modeling, since it has no repercussion on the outcome. The crank has been to be kinematically guided, and since the linkage has only one degree of the freedom, the position, velocity and acceleration of the rest of the mechanism are univocally defined. This fact would be a key point in case of an inverse dynamic analysis, but it was not the objective in previous section.

It has been demonstrate that the prediction about the kinematic behavior of the mechanism adjust significantly to the results obtained by using more sophisticated tools. In opposition to this, a dynamic simulation must be mandatorily carried out with the help of computers. On its basic form, a typical multibody simulation is performed according to the flowchart depicted in Fig. 6.

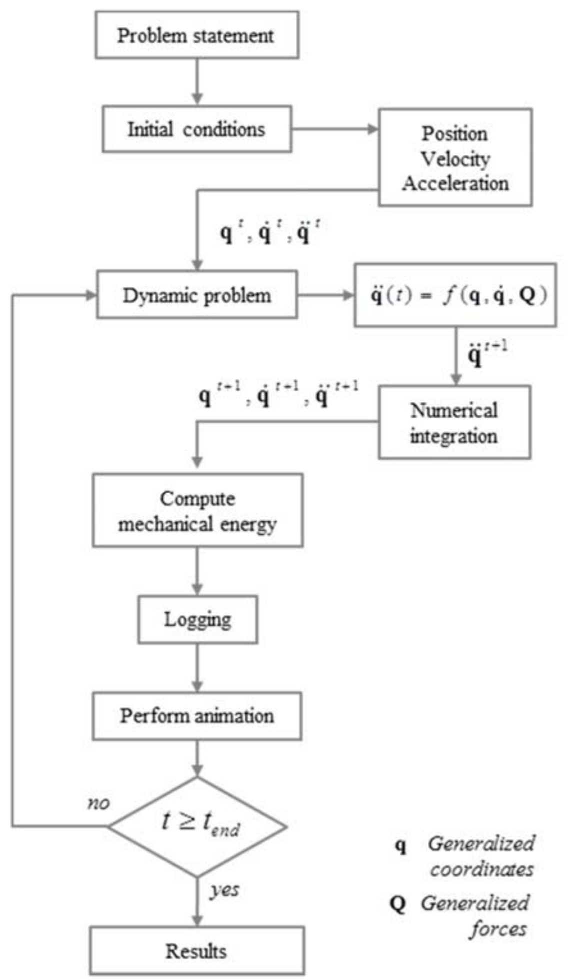
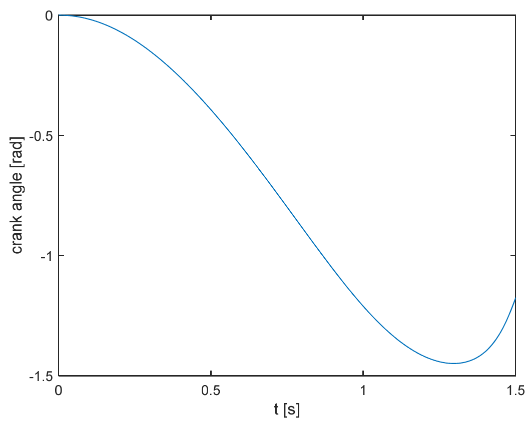
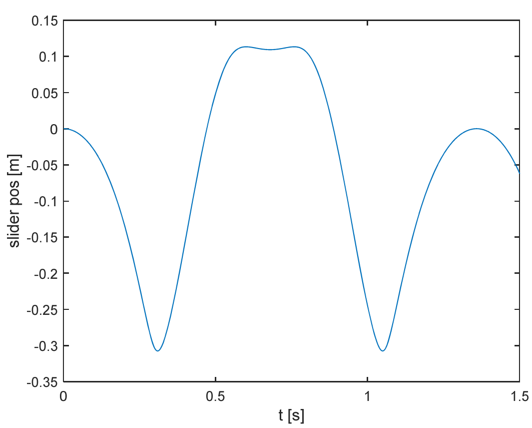


Fig. 6. A schematic flow diagram of dynamic simulations.

For a given initial conditions, simulation starts with the resolution of the kinematic problem. The position problem implies the resolution of a non-linear problem, which is usually performed by an iterative method. As a solution of this problem, a set of generalized coordinates (**q**) which satisfy the position constraints of the mechanism are obtained. Then, the calculation of the generalized velocities ($\dot{\mathbf{q}}$) and accelerations ($\ddot{\mathbf{q}}$) is straightforward. Once the initial kinematic problem is accomplished, the dynamic problem is performed inside the main loop of the simulation. According to the state of the system in a determined instant *t* affected by the actuating generalized forces (**Q**), the acceleration of the system at *t+1* is addressed. At this stage it is necessary to employ a numeric integrator in order to obtain the generalized positions and velocities as such instant. The fourth-order Runge-Kutta explicit integrator or the implicit trapezoidal method may be employed for this purpose. Once the new state is addressed, it is a good practice to compute the energy transformations and dissipations, to check the conservation of the mechanical energy and to record the results. In case of performing an animation, the new frame should be refreshed also at this stage. This process is repeated for each step time until the final simulation time is reached.



a) Results for mechanism 1



b) Results for mechanism 2

Fig. 7. Dynamic simulation: (a) represents the crank angularposition over time for the first mechanism, and (b) the linear position of the slider of the second.

In this subsection, a forward dynamic analysis is addressed by using Simmechanics. Hence, the mechanical parameters of the linkage have an important impact on the results that are evaluated. In this regard, the values of mass assigned to the crank and coupler links 0.9 kg and 0.75 kg respectively in both problems. Furthermore, for the case of the rocker in the first problem, a mass of 1.5 kg has been stated, and the slider body in the second problem has been considered massless. The simulations carried out consist of the observation of both models when are released from the same initial position as depicted in the original images. The only force acting is the gravity force, and the simulation time is 1.5 s. The numeric solver employed is the fourth-order Runge-Kutta explicit integrator. In the first problem, a sensor has been attached to the first revolute joint, which measures de evolution of the angular position of the crank. In the second problem, the sensor is placed at the prismatic joint, which measures the displacement of the slider body. The results of this analysis are depicted in Fig. 7.

V. Conclusions

The paper is aimed to show past mechanism designs with modern features of interest by reanalyzing them with modern means of representing the performance aspects that were the basis of their success. A consideration of historical frames is also presented to properly locate in time and technology the examples that have been reported to show not only the feasibility but also the practical value of such a reconsideration of past mechanism designs.

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